EFFECTS OF WALL PERMITTIVITY AND PLASMA THICKNESS ON THE PLASMA REFLECTION COEFFICIENT

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ABSTRACT. The equation defining the E-field reflection coefficient for a bounded homogeneous plasma slab is solved numerically. Representative computer results are presented which indicate that the magnitude of the reflection coefficient can be maximized by choosing a wall material with the proper permittivity or by choosing the proper plasma width.

INTRODUCTION

It is often necessary to determine the electron density and the collisional frequency of an ionized gas (plasma) in a situation where no physical contact may occur between the measuring device and the plasma. Plasma diagnostic theory and microwave techniques are useful in such cases. The success of these techniques depends upon the existence of a theory which accurately describes the physical situation under consideration and which is amenable to interpretation in terms of experimental measurements. To determine the behavior of a region containing charged particles, Maxwell's equations and the equation of motion of an electron are utilized to effect a solution, (Burke and Crawford, 1964). In the present discussion, ion motion is assumed to have a negligible effect on the problem solution, and it is assumed that no external magnetic field is applied. When the incident electric vector E is perpendicular to the plane of incidence, Shockley and Howe (1965) have shown that E-field reflection coefficient for a semi-infinite homogeneous plasma slab bounded by dielectric walls and probed at oblique incidence by a uniform plane wave may be expressed as

$$\Gamma_R = \Big\{\Big[2\cosh 2\phi_2 + \left(\frac{Z_2}{Z_1} + \frac{Z_1}{Z_2}\right)\sinh 2\phi_2\Big]\cosh\phi_3 + \Big[\Big\{\frac{Z_2}{Z_3} + \frac{Z_3}{Z_2}\Big)\sinh 2\phi_2$$

$$+\left(\frac{Z_1}{Z_3} + \frac{Z_3}{Z_1}\right)\cosh^2\phi_2 + \left(\frac{Z_2^2}{Z_1 Z_3} + \frac{Z_1 Z_3}{Z_2^2}\right)\sinh^2\phi_2\Big]\sinh\phi_3\Big\}$$

$$\Big\{\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\right)\sinh 2\phi_2\cosh\phi_3 + \Big[\left(\frac{Z_3}{Z_1} - \frac{Z_1}{Z_3}\right)\cosh^2\phi_2 \quad \ldots \quad (1)$$

$$+\left(\frac{Z^2_2}{Z_1Z_3}-\frac{Z_1Z_3}{Z^2_2}\sinh{}^2\phi^2\right]\sinh\phi_3\Big\}$$

where θ_1 is the angle of incidence in free space at the interface of the first bounding wall and $Z_1 = 120\pi \sec\theta_1$, $Z_n = (120\,\pi)/\sqrt{K_n - \sin^2\theta_1}$, $(n = 2, 3)$, $\phi_n = jk_0 d_n \sqrt{K_n - \sin^2\theta_1}$, $(n = 2, 3)$, k_0 is the free space propagation constant, the d_n are the widths of the dielectric walls and the plasma, and the K_n are the complex relative permittivities of the dielectric walls and the plasma. K_2 is assumed to be equal to ϵ, the relative dielectric permittivity, implying that the walls are lossless. K_3 may be expressed as

$$K_3 = \left[\left(1-\frac{\alpha^2}{1+\beta^2}-j\left(\frac{\alpha^2\beta}{1+\beta^2}\right)\right]\tag{2}$$

where $\alpha^2 = (\omega_p/\omega)^2$, $\beta^2 = (\nu/\omega)^2$, ω^p is the plasma frequency, ν is the average electron collisional frequency, and ω is the frequency of the propagating wave.

DISCUSSION

It has been noted that the reflection coefficient is a maximum when the container wall widths are approximately $(2n+1)\ \lambda/4$ in optical path width, where $n = 0, 1, 2, \ldots$ and λ is the wavelength in the wall material, (Bachynski and

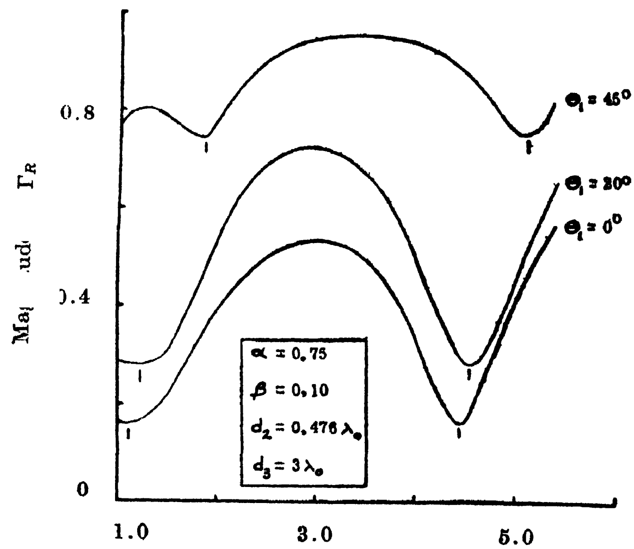

Fig. 1. Variation of the magnitude of Γ_R as a function of the relative permittivity of the walls.

Graf, 1964; and Shockley and Bowie, 1966). The equivalent optical path length of the walls perpendicular to the wall interfaces is given by $d_2\sqrt{\epsilon_r - \sin^2\theta_1}$ and the equivalent wavelength by $\lambda' = \lambda_0/\sqrt{\epsilon_r - \sin^2\theta_1}$. When the wall widths are fixed the permittivity of the walls may be varied to produce an effect equivalent to a change in wall width. The curves in figure 1 indicate a comparison of the variation in the magnitude of the reflection coefficient for normal and oblique incidence. At the points where the equivalent path length is an integral multiple of $\lambda_0/2$, the magnitude of the reflection coefficient returns to its initial value. These points have been marked on the curves. When the wall width and permittivity are fixed and the plasma width is varied, the magnitude of the reflection coefficient can only be maximized within certain limits since the wall characteristics determine the maximum value of Γ_R. However, as d_3 is increased Γ_R varies between relative maximum values as shown in figure 2. In conclusion, a careful choice must be made of the plasma container geometry in an effort to maximize measurement sensitivity.

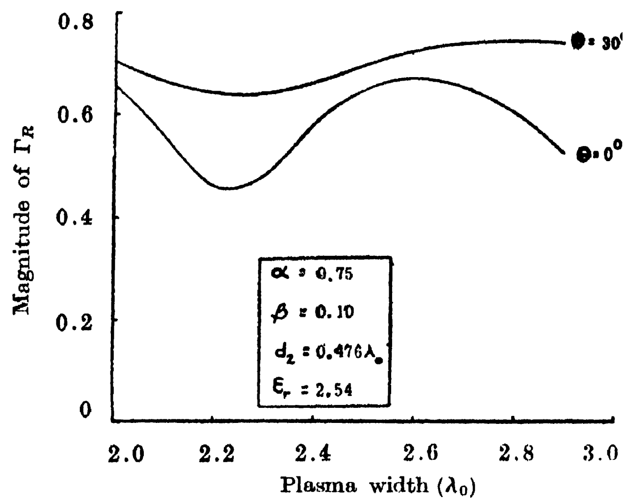

Fig. 2. Variation of the magnitude of Γ_R as a function of the plasma width.

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